

Spin-charge filtering through a spin-orbit coupled quantum dot controlled via an Aharonov-Bohm interferometer

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We show that a strongly correlated quantum dot embedded in an Aharonov-Bohm interferometer can be used to filter both charge and spin at zero voltage bias. The magnitude with which the Aharonov-Bohm arm is coupled to the system controls the many-body effects on the quantum dot. When the quantum dot is in the Kondo regime the flow of charge through the system can be tuned by the phase of the Aharonov-Bohm arm, φ_{AB} . Furthermore when a spin-orbit interaction is present on a Kondo quantum dot we can control the flow of spin by the spin-orbit phase, φ_{SO} . The existence of the Kondo peak at the Fermi energy makes it possible to control the flow of both charge and spin in the zero voltage bias limit.

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I. INTRODUCTION

The ability to easily control charge and spin transport is of great importance in nanotechnology, specifically spintronics. The Quantum Dot Aharonov-Bohm Interferometer (QD-ABI) (Figure 1) has been found to be a candidate for manipulating electron spins^{1,2}. In recent years a number of theories^{3,4} have been put forth to take advantage of the interference effects in such a geometry. Sun and Xie¹ showed that the spin polarization on the QD may be controlled via the voltage bias. In the presence of a local coulomb interaction on the QD Hofstetter *et al.*³ studied the dependence of the Fano line shape on the AB phase.

In this paper we show that the addition of a spin-orbit (SO) interaction on the strongly correlated QD allows the QD-ABI to function as a spin-charge filter. The ability of this system to act as a filter is a consequence of the interference between the continuum (AB arm) and the localized state (QD), which is known as the Fano effect⁵. Aside from the QD-ABI the Fano effect has been observed in a single electron transistor⁶, a quantum wire with a side coupled QD^{7,8}, and multiwall carbon nanotubes in a crossed geometry⁹.

The presence of the AB arm tends to localize electrons on the QD, depending on how strongly the arm is coupled to the system. Therefore this property will give us an effective control over the strength of these correlations on the QD. We may exploit this tuning of the many-body physics to control the charge-spin transport of the QD-ABI system.

The QD interactions we will study in this paper are an on-site Coulomb interaction and the Rashba spin-orbit interaction¹⁰. The Coulomb interaction in the Kondo regime will allow us to filter both charge and spin at zero bias. The reason for this is due to the fact that the Kondo effect induces a sharp resonance in the QD spectral function at the Fermi energy. The further addition of the SO interaction, which is induced by the application of a gate potential, for a single orbital QD, will create a spin dependent phase factor² in the AB arm tunneling

coefficient.

Often the spin-orbit interaction is considered a coupling with spin degrees of freedom mediated by interlevel transitions in quantum dot systems and therefore, due to the significant level spacing and the Coulomb interaction, the inter-level SO coupling strengths in QD systems are thought to be small. However, as pointed out in Ref.², an intra-level phase factor induced by the SO coupling may be realized in high g -factor systems¹¹ such as InGaAs quantum dots. Without the interference effect, *eg.* without the AB-arm, such effect can be ignored. However, the presence of the AB-arm not only manifests the intra-level SO interference effects but also effectively controls the many-body effects on the QD, hence a strong influence on the charge-spin transport.

The experimental setup of the QD-ABI is consistent with that of Kobayashi *et al.*¹². The left and right leads are modeled as infinite non-interacting electron reservoirs. They are connected to each other via two arms. The top arm is the AB arm which has a complex tunneling coefficient $t_0 = |t_0|e^{i\varphi_{AB}}$ that is controlled by a magnetic field. The sign of φ_{AB} is positive for electrons traveling from the left reservoir to the right reservoir. We consider the magnetic field to be small enough so that we may ignore the Zeeman splitting of the QD energy level. The bottom arm contains the embedded QD with real tunneling coefficients t_L and t_R . We choose t_L , t_R to be real because the L , R states are defined to absorb the phase factor. Our calculations are carried out in the low-bias, linear response regime.

II. THEORY

A. Transmission coefficient $T(\epsilon)$

In this section of the paper we will derive the general transport functions for the QD-ABI, which are correct with the interaction on the QD. The non-interacting

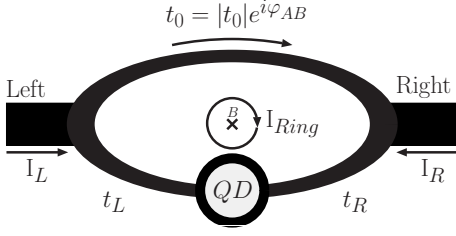


FIG. 1: Experimental setup of the quantum dot embedded AB interferometer. Two infinite electron reservoirs (Left, Right) are connected by two arms. The bottom arm contains the embedded QD with real tunneling coefficients t_L and t_R . The upper arm is a direct connection between the left and right leads with a complex tunneling coefficient $t_0 = |t_0|e^{i\varphi_{AB}}$, where φ_{AB} is a phase factor controlled by the magnetic field, B . The sign of φ_{AB} is positive for electrons traveling from the left to the right, as the arrow indicates. I_L , I_R , and I_{Ring} are the Left, Right, and Ring currents respectively.

Hamiltonian of the system is given by

$$\mathcal{H} = \mathcal{H}_{L,R} + \mathcal{H}_d + \mathcal{H}_t. \quad (1)$$

The Hamiltonian \mathcal{H} consists of three parts: $\mathcal{H}_{L,R}$ describes the reservoirs, \mathcal{H}_d the QD, and \mathcal{H}_t the tunneling between the reservoirs.

$$\mathcal{H}_{L,R} = \sum_{\alpha k \sigma} \epsilon_{\alpha k} c_{\alpha k \sigma}^\dagger c_{\alpha k \sigma} \quad (2)$$

$$\mathcal{H}_d = \sum_{\sigma} \epsilon_d d_{\sigma}^\dagger d_{\sigma} \quad (3)$$

$$\mathcal{H}_t = -\frac{1}{\sqrt{\Omega}} \sum_{\alpha k \sigma} t_{\alpha} (c_{\alpha k \sigma}^\dagger d_{\sigma} + h.c.) - \frac{1}{\Omega} \sum_{k, k'} (t_0 c_{Lk\sigma}^\dagger c_{Rk'\sigma} + h.c.), \quad (4)$$

where $c_{\alpha k \sigma}^\dagger$ ($c_{\alpha k \sigma}$) and d_{σ}^\dagger (d_{σ}) are the creation (annihilation) operators with momentum k and spin σ of the $\alpha = (L, R)$ reservoir and the QD, respectively. In addition, Ω is the volume of the reservoirs which is taken to infinity.

Before presenting the Landauer formula for the current we define the parameters. In the non-interacting

limit without the AB arm, the line broadening of the QD spectral function due to the leads is $\Gamma = \Gamma_L + \Gamma_R$ where $\Gamma_{\alpha} = \pi N_0 t_{\alpha}^2$. In our calculations we take the density of states, N_0 , to be a constant.

The exact current from the L to R reservoir, regardless of the local interaction on the QD, is given by the Landauer formula¹³,

$$I_L = \frac{2e^2}{h} \int_{-\infty}^{\infty} T(\epsilon) \Delta f(\epsilon) d\epsilon. \quad (5)$$

Here $\Delta f(\epsilon) = f_L(\epsilon) - f_R(\epsilon)$ and $T(\epsilon)$ is the transmission function. The transmission function was previously reported in Refs.^{3,4}, although without derivation. Therefore we present the derivation, which makes use of standard Keldysh Green function techniques^{14,15}, in the Appendix. Here we summarize the results.

The transmission function may first be decoupled into two parts, the flow of current through the QD and through the AB arm,

$$T(\epsilon) \Delta f = i_{QD}(\epsilon) + i_{AB}(\epsilon) \quad (6)$$

$$i_{QD}(\epsilon) = -t_L (G_{dL}^<(\epsilon) - G_{Ld}^<(\epsilon)) = 2\text{Re}[-t_L G_{dL}^<(\epsilon)]$$

$$i_{AB}(\epsilon) = [-t_0^* G_{RL}^<(\epsilon) + t_0 G_{LR}^<(\epsilon)] = 2\text{Re}[-t_0^* G_{RL}^<(\epsilon)],$$

where we have used the relation, $G_{\alpha\beta}^< = -(G_{\beta\alpha}^<)^*$. i_{QD} and i_{AB} are the contributions to the current from the QD and the AB arm respectively. To simplify our notation we define the following parameters

$$T_0 = \frac{4r_0}{(1+r_0)^2} \quad (7)$$

$$R_0 = 1 - T_0 = \left[\frac{1-r_0}{1+r_0} \right]^2 \quad (8)$$

$$\alpha = \frac{4\Gamma_L \Gamma_R}{\Gamma^2} \quad (9)$$

$$\bar{\Gamma} = \frac{\Gamma}{1+r_0}, \quad (10)$$

where $r_0 = \pi^2 N_0^2 |t_0|^2$. Here T_0 is the transmission function when the QD is disconnected from the left and right reservoirs, i.e. $t_L = t_R = 0$. The current through the QD and AB arm is found below.

$$i_{QD} = \left[-\alpha \bar{\Gamma} \sqrt{R_0} - 2T_0 \Gamma_L \Gamma_R \sin^2(\varphi_{AB}) - \frac{\Gamma_L - \Gamma_R}{2} \sqrt{\alpha T_0} \sin(\varphi_{AB}) \right] \text{Im}[G_{dd}^R] \Delta f - \bar{\Gamma} \sqrt{\alpha T_0} \cos(\varphi_{AB}) \text{Re}[G_{dd}^R] \Delta f - i_{Ring} \quad (11)$$

$$i_{AB} = \left[\alpha \bar{\Gamma} \sqrt{T_0} \left(\sqrt{T_0} \cos^2(\varphi_{AB}) - 1 \right) + 2T_0 \Gamma_L \Gamma_R \sin^2(\varphi_{AB}) + \frac{\Gamma_L - \Gamma_R}{2} \sqrt{\alpha T_0} \sin(\varphi_{AB}) \right] \text{Im}[G_{dd}^R] \Delta f + 2T_0 \bar{\Gamma} \sqrt{\alpha} \cos(\varphi_{AB}) \text{Re}[G_{dd}^R] \Delta f + i_{Ring} + T_0 \Delta f. \quad (12)$$

In our analysis we find there exists a ring current even at zero bias,

$$i_{\text{Ring}} = -\sqrt{\alpha T_0} \Gamma \sin \varphi_{AB} \text{Im}[G_{dd}^R] \bar{f}. \quad (13)$$

Here $\bar{f} = (f_L + f_R)/2$ is the average Fermi function. This current flows in the clockwise direction through the AB ring and persists even at zero bias since it is proportional to \bar{f} , not Δf . Although the ring current can be of the same order of magnitude as the total current, it does not contribute to the source-drain current.

Using Eq. (6) we arrive at the exact transmission function,

$$T(\epsilon) = T_0 - 2\bar{\Gamma} \sqrt{\alpha T_0 R_0} \cos(\varphi_{AB}) \text{Re}[G_{dd}^R(\epsilon)] - \bar{\Gamma} [\alpha(1 - T_0 \cos^2(\varphi_{AB})) - T_0] \text{Im}[G_{dd}^R(\epsilon)]. \quad (14)$$

We emphasize that $G_{dd}^R(\epsilon)$ is the full interacting QD retarded Green function, and Eq. 14 applies to systems with an interacting QD.

B. Non-interacting limit

In the non-interacting limit we will address two important points. First, as the coupling to the left and right reservoirs is increased through $|t_0|$, the electron becomes more localized on the QD. Secondly, the noninteracting system is not suitable for controlling both charge and spin transport.

The noninteracting QD spectral function is given by

$$A(\epsilon) = \frac{\bar{\Gamma}/\pi}{(\epsilon - \epsilon_d - \delta)^2 + \Gamma^2}, \quad (15)$$

where

$$\delta = \frac{2\sqrt{r_0 \Gamma_L \Gamma_R}}{1 + r_0} \cos(\varphi_{AB}). \quad (16)$$

and $\bar{\Gamma}$ was defined in Eq. (10). We plot $A(\epsilon)$ in Figure 2 for different values of $|t_0|$. Notice that as the magnitude of $|t_0|$ is increased, the QD spectral function becomes sharper and the center of the peak is shifted. This shift in the QD energy, δ and the reduced line broadening, $\bar{\Gamma}$ are easily understood by performing a bonding-anti-bonding transformation of the leads in real space. Upon doing so the QD is now only coupled to the first site of the bonding chain where the local energy of this site is shifted by $-|t_0| \cos(\varphi_{AB})$. Thus by increasing $|t_0|$, the connection of the QD to the bonding chain is reduced, and the energy level of the QD is shifted upward.

The differential conductance is given as $G(\Phi) = e(dI/d\Phi)$, and in the zero bias and low-temperature limit $G(0) = \frac{2e^2}{h} T(0)$. Therefore at equilibrium we only need $T(0)$ to determine the conductance. In Figure 3 we plot the transmission amplitude as a function of the AB phase, φ_{AB} . When $\varphi_{AB} = \pm \frac{\pi}{2}$, $T(0) = 1$. When $\varphi_{AB} = (0, \pi)$,

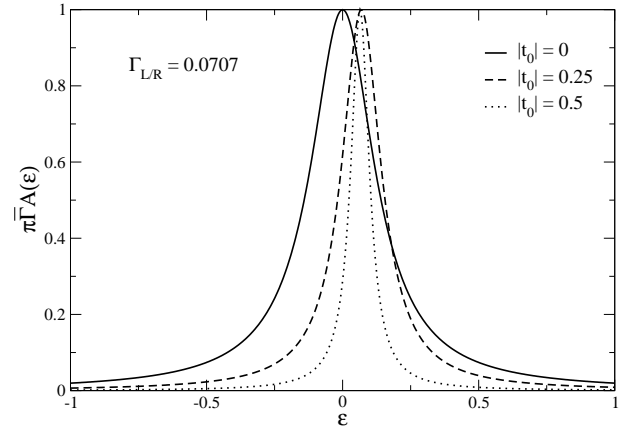


FIG. 2: QD spectral function for different magnitudes of $|t_0|$ at $\varphi_{AB} = 0$. As $|t_0|$ is increased the electron becomes more localized on the QD and the distribution of energies shift away from the Fermi energy.

the Fano anti-resonance becomes most prominent and the minimum of the peaks are positioned significantly above and below the Fermi energy. As a result of this fact we are not able to extinguish the charge or spin conductance at zero bias. If we were able to shift the minima of these anti-resonance peaks to the Fermi energy, then we would be able to fully control the transport through this system at zero bias by tuning φ_{AB} . In the next section of this paper we show that we can accomplish this through the

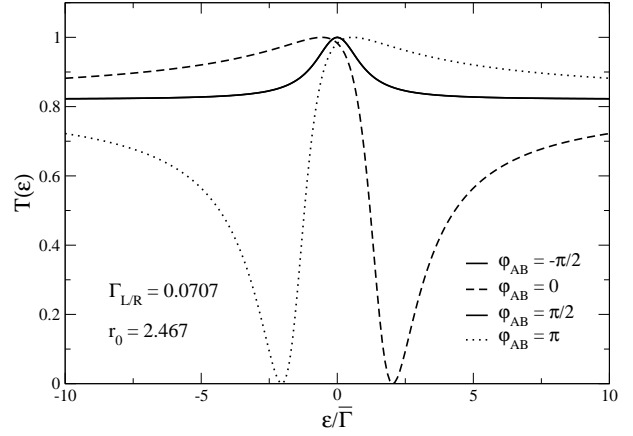


FIG. 3: Noninteracting transmission function for different values of φ_{AB} . When $\varphi_{AB} = (-\frac{\pi}{2}, \frac{\pi}{2})$ we see a resonance at the QD energy level. The $\varphi_{AB} = (-\frac{\pi}{2}, \frac{\pi}{2})$ curves coincide, away from these curves the interference effects become more pronounced and the transmission function becomes asymmetrical. When $\varphi_{AB} = (0, \pi)$ the interference effects become most pronounced and a very strong anti-resonance emerges which is shifted to the left or right of the QD energy level. In the limit $|\epsilon| \rightarrow \infty$ the transmission converges to a finite value, due to the AB arm.

C. Interacting Fano Effect

When a local many-body interaction is present on the QD site, the problem is essentially reduced to calculating the full QD retarded Green function, $G_{dd}^R(\epsilon)$. Once we know this Green function we may then calculate the transmission function using Eq. (14). In this paper we take into account the Coulomb interaction on the QD in the form of the Anderson interaction,

$$\mathcal{H}_{int} = U \hat{n}_{d\uparrow} \hat{n}_{d\downarrow}. \quad (17)$$

We perform the diagrammatic calculation in the imaginary-time formalism. The QD Green function, at imaginary Matsubara frequency $i\omega_n = i\frac{(2n+1)\pi}{\beta}$, is given by $G_{dd}(i\omega_n) = [(G_{dd}^0)^{-1}(i\omega_n) - \Sigma(i\omega_n)]^{-1}$, where the self energy, $\Sigma(i\omega_n)$ is calculated to second-order in U (Figure 4). The second-order perturbation theory has been studied extensively^{17,18,19,20,21} and shown to be a very good approximation in the particle-hole symmetric limit up to values of $U/\bar{\Gamma} \simeq 6$ (weak-coupling regime). This weak-coupling approximation has also been used within the framework of dynamical mean field theory²², and produced agreement with nonperturbative methods. In our model the particle-hole symmetry will be broken for $\delta \neq 0$, but the calculation of the zero bias conductance is in excellent agreement with the nonperturbative numerical renormalization group³(NRG) as will be shown.

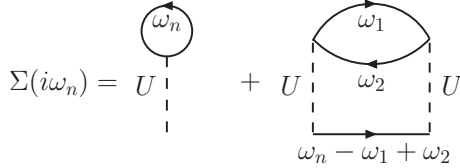


FIG. 4: Self energy expanded to second order in U .

The first-order diagram becomes $U\langle n_d \rangle = \frac{U}{2}$ in the half-filled limit. Absorbing the first-order diagram into the non-interacting Matsubara Green function we have

$$G_{dd}^0(i\omega_n) = \frac{1}{i\omega_n - \delta + i\bar{\Gamma} \cdot \text{Sign}(\omega_n)}, \quad (18)$$

and the second order self energy becomes

$$\Sigma^{(2)}(i\omega_n) = \frac{U^2}{\beta^2} \sum_{\omega_1, \omega_2} G_{dd}^0(i\omega_n - i\omega_1) G_{dd}^0(i\omega_2) G_{dd}^0(i\omega_1 + i\omega_2). \quad (19)$$

Now we are in position to solve this problem numerically. First we calculate the self-energy in Matsubara frequency. Then in order to calculate $T(\epsilon)$ we must analytically continue the Green function to its retarded form in real frequency space, $G_{dd}(i\omega_n \rightarrow \epsilon + i\eta)$. For the numerical analytical continuation we used the N -Point Padé approximant method²³.

Let us first look at the spectral function, $A(\epsilon) = -\frac{1}{\pi} \text{Im}[G_{dd}(\epsilon)]$. For the resonant case ($\varphi_{AB} = \frac{\pi}{2}$) the

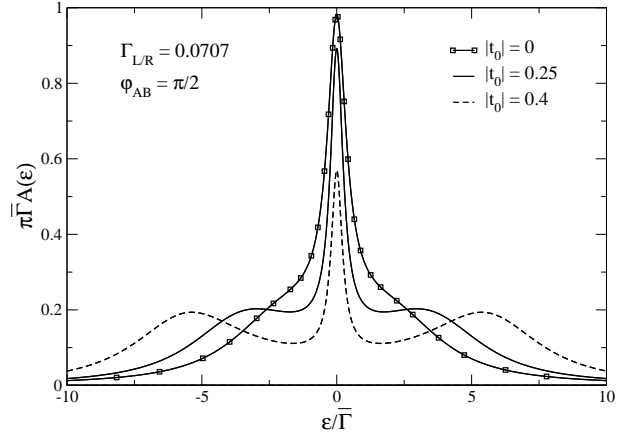


FIG. 5: Interacting QD spectral function for different magnitudes of $|t_0|$ when $\varphi_{AB} = \frac{\pi}{2}$, $U = 0.6$ and $\beta = 160$. We see that as $|t_0|$ is increased the electron becomes more localized on the QD and as a result the correlations due to the Coulomb interaction become more pronounced.

spectral function is plotted for different values of $|t_0|$ in Figure 5. We see that as we increase the coupling $|t_0|$ of the L, R states to the AB arm, the effective many-body interaction $U/\bar{\Gamma}$ is strongly enhanced. From our analysis of the noninteracting system this is exactly what we expected to happen. When $|t_0| = 0$ we start in the weakly interacting valence fluctuating regime, but as we increase $|t_0|$ to 0.4 the Kondo at zero frequency and the Hubbard

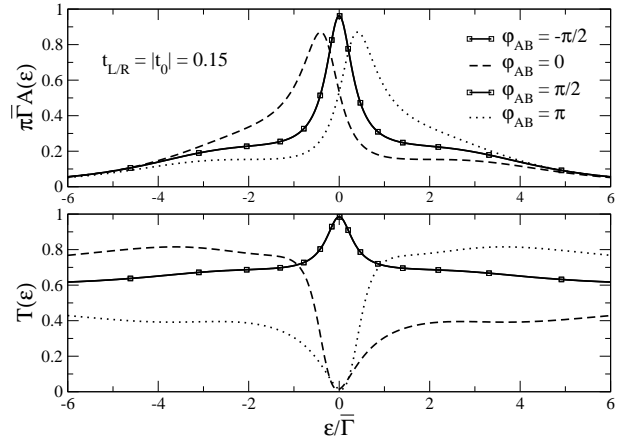


FIG. 6: QD spectral function and transmission amplitude for $U = 0.6$ and $\beta = 160$. The $\varphi_{AB} = \frac{\pi}{2}, -\frac{\pi}{2}$ curves are identical, and at these values the spectral function is symmetric and the transmission amplitude displays resonant behavior. On the other hand when $\varphi_{AB} = 0, \pi$ the spectral function is asymmetric and the transmission amplitude has an anti-resonance.

The transmission functions is given in Figure 6. As in the noninteracting case we have resonance (anti-resonance) phenomena at $\varphi_{AB} = -\frac{\pi}{2}, \frac{\pi}{2}$ ($0, \pi$) respectively. The anti-resonance peaks are known as the Fano-Kondo anti-resonance and have been observed experi-

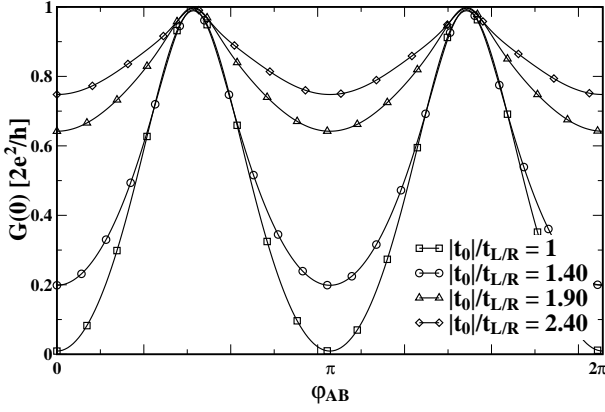


FIG. 7: Zero bias conductance as a function of φ_{AB} for $t_L = t_R = 0.15$, $U = 0.5$ and $\beta = 160$. The conductance approaches unity when $\varphi_{AB} = \frac{2n+1}{2}\pi$. Conversely the conductance approaches a minimum when $\varphi_{AB} = n\pi$, with integer n .

mentally in a quantum wire with a side coupled QD^{7,8}. For $\varphi_{AB} = 0, \pi$ the anti-resonance peaks are antisymmetric of one another about the Fermi energy.

Now let us examine the zero bias conductance. Our results are given in Figure 7 as a function of φ_{AB} for different values of the ratio $|t_0|/t_{L/R}$. The NRG calculation of Ref.³ examined the zero bias conductance as a function of φ_{AB} for different values of the gate potential, ϵ_d . A careful straightforward identification of the model parameters shows that our results agree excellently with the NRG²⁵, which justifies our self-energy approximation. Since $G(0)$ is only dependent upon the value of the Green function at the Fermi energy, this implies that the second-order self energy approximation at least produces reliable results near the Fermi energy for $\delta \neq 0$.

From Figure 7 we see that $G(0)$ oscillates as a function of φ_{AB} and the magnitude of these oscillations is strongly dependant on the ratio, $|t_0|/t_{L/R}$. We find that $|t_0|/t_{L/R} = 1$ is a special case which maximizes the magnitude of the AB oscillations. These oscillations are a consequence of the $\cos^2(\varphi_{AB})$ term in Eq. (14) and the sharp feature near the Fermi energy of the spectral function (Figure 6). The resonance ($G(0) = \max$) [anti-resonance ($G(0) = \min$)] peaks occur at $\varphi_{AB} = \frac{2n+1}{2}\pi$ [$n\pi$], with integer n . Thus by tuning φ_{AB} we may filter the charge through the system at zero voltage bias. The reason why we have so much control over the conductance at zero bias is due to the Kondo effect. The Kondo effect induces a sharp peak near the Fermi energy in the spectral function, insensitive to φ_{AB} . This feature is due to the many body effect and is not present in the non-interacting case (Figure 3).

D. Spin Transport

For a single orbital QD the addition of the spin-orbit interaction induces a spin dependent phase in t_R ², i.e.

$t_R \rightarrow t_R e^{-i\sigma\varphi_{SO}}$, where $\sigma = +(-)$ for spin up(down). Unlike φ_{AB} which arises from the orbital motion, φ_{SO} depends on the spin. To simplify our Hamiltonian we make the unitary transformation

$$c_{Rk}^\dagger \rightarrow e^{is\varphi_{SO}} c_{Rk}^\dagger \quad (20)$$

so that

$$t_0 \rightarrow e^{is\varphi_{SO}} t_0 \quad (21)$$

Now we define a spin dependent AB tunneling coefficient

$$t_{0\sigma} = |t_0| e^{i(\varphi_{AB} + \sigma\varphi_{SO})}. \quad (22)$$

As a result of the spin dependence in $t_{0\sigma}$, the Green's functions also gain a spin dependence and the spin dependant second order self energy is given by

$$\begin{aligned} \Sigma_\sigma^{(2)}(i\omega_n) = & \frac{U^2}{\beta^2} \sum_{\omega_1, \omega_2} G_{dd\sigma}^0(i\omega_n - i\omega_1) \times \\ & G_{dd-\sigma}^0(i\omega_2) G_{dd-\sigma}^0(i\omega_1 + i\omega_2) \end{aligned} \quad (23)$$

Now let us examine the spectral and transmission functions given in Figure 8. As in the spin independent case we see resonance and antiresonance behavior in the transmission function. More importantly, due to the SO interaction we now have two phase factors, φ_{AB} and φ_{SO} , which along with the Kondo resonance near the Fermi energy, we may use to filter the spin up and spin down electrons independently.

Let us take a closer look at Figure 8. We see that when $\varphi_{AB} = \varphi_{SO} = \frac{\pi}{4}$, A_\uparrow has a spectral structure similar to that of a Kondo dot while A_\downarrow develops a slight asymmetry from the spin up case. Looking at the transmission function, T_\uparrow is similar in shape to its spectral function; and T_\uparrow shows strong antiresonance behavior due to the AB ring. This mechanism gives us a strong control on the spin-transport. At the Fermi energy $T_\uparrow(0) \cong 1$ and $T_\downarrow \cong 0$. When $\varphi_{SO} = \frac{\pi}{8}$ both T_\uparrow and T_\downarrow show strong interference effects.

To look at the zero bias spin conductance we turn to Figure 9 where we show the SO oscillations of the zero bias conductance. The behavior of the spin up and spin down conductance is a result of the $\cos^2(\varphi_{AB} + \varphi_{SO})$ term for spin up and the $\cos^2(\varphi_{AB} - \varphi_{SO})$ term for spin down in Eq. (14). In the figure we see that when $\varphi_{AB} = \frac{\pi}{4}$ the spin polarized conductance [$\eta = (G_\uparrow(0) - G_\downarrow(0))/(G_\uparrow(0) + G_\downarrow(0))$] is maximized. Conversely when $\varphi_{SO} = \frac{\pi}{2}$ the spin polarization is suppressed.

The most important results of this paper are shown in the spin polarized conductance in Figure 10. Here we plot η as a function of φ_{SO} for different values of φ_{AB} . The maximum in the spin up/down conductance occurs approximately when $\varphi_{AB} \pm \varphi_{SO} = \frac{2n+1}{2}\pi$, while the minimum occurs at $\varphi_{AB} \pm \varphi_{SO} = m\pi$. This can be seen from the $\cos^2(\varphi)$ factor in Eq. (14), since at the Kondo resonance ($\epsilon \approx 0$) only T_0 and the term proportional to $\text{Im}[G_{dd}^R(\epsilon)]$ become relevant. Therefore the necessary conditions on φ_{SO} and φ_{AB} for the spin polarized

conductance to be maximized and minimized are

$$\left(\frac{\varphi_{SO}}{\varphi_{AB}}\right)_{\eta \approx 1} = \frac{n - m + \frac{1}{2}}{n + m + \frac{1}{2}} \quad (24)$$

$$\left(\frac{\varphi_{SO}}{\varphi_{AB}}\right)_{\eta \approx 0} = \frac{n - m}{n + m + 1}. \quad (25)$$

Here again we emphasize the crucial role which the many-body interactions play in this system. The Kondo effect serves to simultaneously pin the resonance/antiresonance peaks of the spin dependent conductance at the chemical potential, as seen in Figure 6. As a result, when the SO interaction is turned on, we gain complete control over the spin polarization (Figure 10). Furthermore, even if there only exists a small SO interaction in the QD, this device serves to enhance those inherent SO effects. In contrast to this, in the noninteracting device (Figure 3) the resonance/antiresonance peaks occur at significantly different energies, making it impractical to use as a spin/charge filter. Therefore we have shown that the QD-ABI is an ideal spin filter where

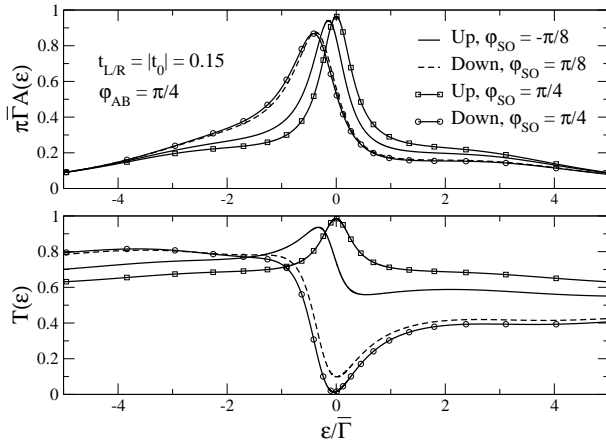


FIG. 8: Spin dependant spectral and transmission functions for $\varphi_{SO} = \frac{\pi}{8}, \frac{\pi}{4}$, $U = 0.6$ and $\beta = 160$. For $\varphi_{SO} = \frac{\pi}{4}$ $A_{\uparrow}(A_{\downarrow})$ is symmetric(asymmetric), while $T_{\uparrow}(T_{\downarrow})$ displays resonance(anti-resonance) behavior. In addition, $T_{\uparrow}(0) \sim 1$ and $T_{\downarrow}(0) \sim 0$. For $\varphi_{SO} = \frac{\pi}{8}$ both A_{\uparrow} and A_{\downarrow} are asymmetrical and A_{\downarrow} lies on the curve of $A_{\uparrow}(\varphi_{SO} = \frac{\pi}{4})$. T_{\uparrow} and T_{\downarrow} both display anti-resonant phenomena.

III. CONCLUSION

In this paper we have shown that the QD-ABI may be used to localize and delocalize electrons on the QD. As a result, when the QD is strongly correlated we can effectively control the strength of these correlations through the magnitude of t_0 . With larger t_0 , electrons get more localized on the QD. In the case of a coulomb interaction on the QD, the Kondo effect induces a sharp peak in the spectral function near the Fermi energy. We emphasize

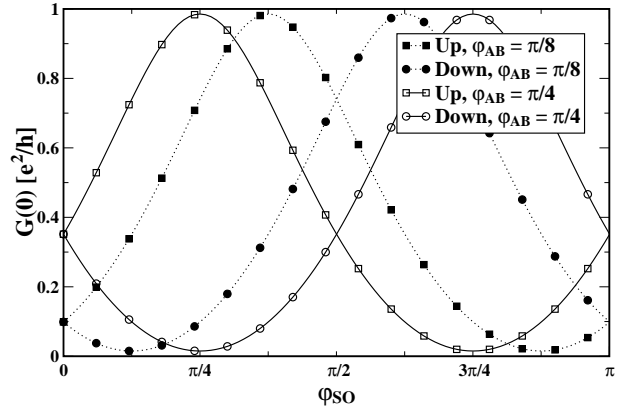


FIG. 9: SO oscillations for $\varphi_{AB} = \frac{\pi}{8}, \frac{\pi}{4}$, $U = 0.6$, and $\beta = 160$. The minima and maxima of the SO oscillations approach 0 and 1 respectively.

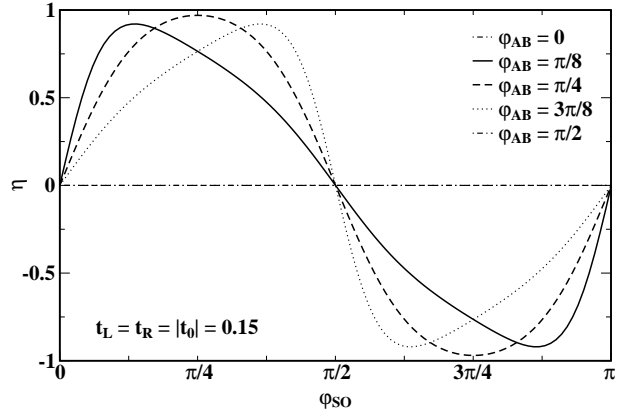


FIG. 10: Spin-polarized conductance for different AB phases when $U = 0.6$ and $\beta = 160$. Here the spin polarization is defined as, $\eta = [G_{\uparrow}(0) - G_{\downarrow}(0)]/[G_{\uparrow}(0) + G_{\downarrow}(0)]$. We see that the spin polarization may be fully controlled by tuning φ_{AB} and φ_{SO} . The $\varphi_{AB} = 0, \frac{\pi}{2}$ curves are identical.

that the exact relation for the transmission, Eq. (14), shows that the resonance/anti-resonance is driven by the AB phase factor once the spectral function develops a sharp feature near the Fermi energy, independent of its many-body character.

The most important result of this paper is realized when a SO interaction is present on the QD in addition to the coulomb interaction. In this case the Kondo peak and the spin dependent $\cos^2(\varphi)$ term induce the resonance/anti-resonance in the zero bias spin dependant conductance. Due to this property we can vary the $G_{\uparrow}(0)$ and $G_{\downarrow}(0)$ between 0 and e^2/h . Further, the SO phase gives us another degree of freedom, in addition to the AB phase, so that the spin up and spin down conductances may be varied independently. Thus by tuning φ_{AB} and φ_{SO} the spin polarization may be fully controlled in the QD-ABI.

APPENDIX A: DERIVATION OF $T(\epsilon)$

In this appendix we present the full derivation of the transmission function. The transmission is given in terms of Keldysh Green functions,

$$T(\epsilon)\Delta f = i_{\text{QD}}(\epsilon) + i_{\text{AB}}(\epsilon) \quad (\text{A1})$$

$$i_{\text{QD}}(\epsilon) = -t_L[G_{dL}^<(\epsilon) - G_{Ld}^<(\epsilon)] \quad (\text{A2})$$

$$i_{\text{AB}}(\epsilon) = [-t_0^*G_{RL}^<(\epsilon) + t_0G_{LR}^<(\epsilon)], \quad (\text{A3})$$

where $G_{dL}^<$ and $G_{RL}^<$ are the Fourier transforms of the following time dependent nonequilibrium Green functions (NEGFs),

$$G_{dL}^<(t) = \frac{1}{\sqrt{\Omega}} \sum_k i \langle d^\dagger(0) c_{Lk}(t) \rangle \quad (\text{A4})$$

$$G_{RL}^<(t) = \frac{1}{\Omega} \sum_{kk'} i \langle c_{Lk}^\dagger(0) c_{Lk'}(t) \rangle, \quad (\text{A5})$$

and $G_{Ld}^< = -(G_{dL}^<)^*$ and $G_{LR}^< = -(G_{RL}^<)^*$. In calculating these Green functions it becomes convenient to disconnect the QD from the reservoirs, i.e. $t_L = t_R = 0$,

and define the following QD-excluded Green functions

$$g_{LL}^r = g_{RR}^r = \frac{-i\pi N_0}{1+r_0} \quad (\text{A6})$$

$$g_{RL}^r = (g_{LR}^r)^* = \frac{\pi^2 N_0^2 t_0}{1+r_0} \quad (\text{A7})$$

$$g_{LL}^< = \frac{2\pi i N_0}{(1+r_0)^2} (f_L + r_0 f_R) \quad (\text{A8})$$

$$g_{RR}^< = \frac{2\pi i N_0}{(1+r_0)^2} (f_R + r_0 f_L) \quad (\text{A9})$$

$$g_{RL}^< = -(g_{LR}^<)^* = \frac{-2\pi^2 N_0^2}{(1+r_0)^2} t_0 (f_L - f_R). \quad (\text{A10})$$

We express $G_{dL}^<$ and $G_{RL}^<$ in terms of the fully interacting retarded and advance QD Green functions and the above QD-excluded noninteracting NEGFs.

To simplify our notation we define $F_{\alpha d}^r$, $F_{\alpha d}^a$, and $F_{\alpha d}^<$ which are the retarded, advanced, and lesser Green functions which describe the transport from the QD to the α lead in terms of the QD-excluded Green functions.

$$F_{Ld}^r = (F_{dL}^a)^* = t_L (t_L g_{LL} + t_R g_{LR})^r = \frac{1}{1+r_0} \left(-i\Gamma_L + \sqrt{r_0 \Gamma_L \Gamma_R} e^{-i\varphi_{AB}} \right) \quad (\text{A11})$$

$$F_{Rd}^r = (F_{dR}^a)^* = t_R (t_R g_{RR} + t_L g_{RL})^r = \frac{1}{1+r_0} \left(-i\Gamma_R + \sqrt{r_0 \Gamma_L \Gamma_R} e^{i\varphi_{AB}} \right) \quad (\text{A12})$$

$$F_{Ld}^< = -(F_{dL}^<)^* = t_L (t_L g_{LL} + t_R g_{LR})^< = \frac{2}{1+r_0} \left[i\Gamma_L (f_L + r_0 f_R) + \sqrt{r_0 \Gamma_L \Gamma_R} e^{-i\varphi_{AB}} (f_L - f_R) \right] \quad (\text{A13})$$

$$F_{Rd}^< = -(F_{dR}^<)^* = t_R (t_R g_{RR} + t_L g_{RL})^< = \frac{2}{1+r_0} \left[i\Gamma_R (f_R + r_0 f_L) + \sqrt{r_0 \Gamma_L \Gamma_R} e^{i\varphi_{AB}} (f_R - f_L) \right]. \quad (\text{A14})$$

Therefore G_{RL} and G_{dL} may be written in terms of F 's and the full QD Green function as

$$t_L t_R G_{RL} = F_{Rd} G_{dd} F_{dL} \quad (\text{A15})$$

$$-t_L G_{dL} = G_{dd} F_{dL}. \quad (\text{A16})$$

Using the Keldysh Green function relations²⁶

$$(AB)^< = A^< B^a + A^r B^< \quad (\text{A17})$$

$$(ABC)^< = A^< B^a C^a + A^r B^< C^a + A^r B^r C^<, \quad (\text{A18})$$

the lesser Green functions become

$$G_{RL}^< = F_{Rd}^< G_{dd}^a F_{dL}^a + F_{Rd}^r G_{dd}^< F_{dL}^a + F_{Rd}^< G_{dd}^a F_{dL}^a \quad (\text{A19})$$

$$-t_L G_{dL}^< = G_{dd}^< F_{dL}^a + G_{dd}^r F_{dL}^<. \quad (\text{A20})$$

Making use of the nonequilibrium steady state condition, $i_L + i_R = 0$, we may construct $G_{dd}^<$ in terms of G_{dd}^r and G_{dd}^a as follows. The ensemble averaged currents are given by

$$i_\alpha = -t_\alpha (G_{d\alpha}^< - G_{\alpha d}^<) = (F_{d\alpha}^a - F_{\alpha d}^r) G_{dd}^< + G_{dd}^r F_{d\alpha}^< + F_{\alpha d}^< G_{dd}^a \quad (\text{A21})$$

where $\alpha = (L, R)$. Therefore by invoking the steady state condition the QD lesser Green function becomes

$$G_{dd}^< = \frac{(F_{dL}^< + F_{dR}^<) G_{dd}^r - (F_{Ld}^< + F_{Rd}^<) G_{dd}^a}{(F_{Ld}^r - F_{dL}^a) + (F_{Rd}^r - F_{dR}^a)}. \quad (\text{A22})$$

Inserting $G_{dd}^<$ into Eq. (A21) with $\alpha = L$ we arrive at i_{QD} , Eq. (11). The current through the AB arm is given by

$$\begin{aligned}
t_L t_R i_{AB} &= t_L t_R (-t_0^* G_{RL}^< + t_0 G_{LR}^<) \\
&= (-t_0^* F_{Rd}^r F_{dL}^< + t_0 F_{Ld}^r F_{dR}^<) G_{dd}^r + (-t_0^* F_{Rd}^< F_{dL}^a + t_0 F_{Ld}^< F_{dR}^a) G_{dd}^a + \\
&\quad (-t_0^* F_{Rd}^r F_{dL}^a + t_0 F_{Ld}^r F_{dR}^a) G_{dd}^<,
\end{aligned} \tag{A23}$$

where all of the terms have been solved for. Thus after doing some algebra, i_{AB} may be expressed as Eq. (12).

Using Eq. (6), we arrive at the transmission function, Eq. (14).

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